Argomento 7

- Lezione 11
- Lezione 12
- Lezione 13

Francesca Apollonio

Dipartimento Ingegneria Elettronica

E-mail: apollonio@die.uniroma1.it

Riflessione e rifrazione: incidenza obliqua

Quando l'onda incidente si propaga in direzione obliqua rispetto all'asse z le onde riflesse e trasmesse hanno direzioni di propagazione e polarizzazioni diverse da quelle dell'onda incidente.

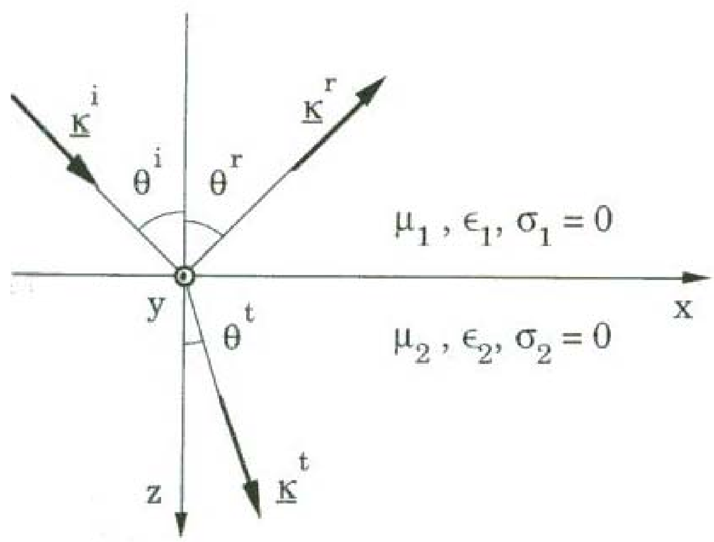


Fig. V.4

$$\underline{\mathbf{k}}^{i} = k_1 \underline{\beta}_0^i = k_x^i \underline{\mathbf{x}}_0 + k_z^i \underline{\mathbf{z}}_0$$

$$\begin{cases} k_x^i = \underline{\mathbf{k}}^i \cdot \underline{\mathbf{x}}_0 = k_1 \sin\theta^i \\ k_z^i = \underline{\mathbf{k}}^i \cdot \underline{\mathbf{z}}_0 = k_1 \cos\theta^i \end{cases}$$

apollonio@die.uniroma1.it

$$\underline{\mathbf{E}}^i(x,z)=\underline{\mathbf{E}}_0^i e^{-j\left(k_x^i x+k_z^i z\right)}$$

$$\underline{\mathbf{H}}^i(x,z)=\underline{\mathbf{H}}_0^i e^{-j\left(k_x^i x+k_z^i z\right)}$$

Per la condizione di continuità dei campi elettrico e magnetico tangenziali in z=0

$$\underline{\mathbf{z}}_0\times\left\{\underline{\mathbf{E}_0}^t e^{-j\left(k_x{}^t x+k_y{}^t y\right)}-\left[\underline{\mathbf{E}}_0^i e^{-j\left(k_x^i x\right)}+\underline{\mathbf{E}_0}^r e^{-j\left(k_x{}^r x+k_y{}^r y\right)}\right]\right\}=0$$

$$\underline{\mathbf{z}}_0\times\left\{\underline{\mathbf{H}_0}^t e^{-j\left(k_x{}^t x+k_y{}^t y\right)}-\left[\underline{\mathbf{H}}_0^i e^{-j\left(k_x^i x\right)}+\underline{\mathbf{H}_0}^r e^{-j\left(k_x{}^r x+k_y{}^r y\right)}\right]\right\}=0$$

$$\Downarrow$$

$$k_x{}^r=k_x{}^i \qquad k_y{}^r=0$$

$$k_x{}^t=k_x{}^i \qquad k_y{}^t=0$$

apollonio@die.uniroma1.it

$$\underline{\mathbf{k}}^r = k_1 \underline{\beta_0^r} = k_x^r \underline{\mathbf{x}}_0 + k_z^r \underline{\mathbf{z}}_0$$

$$\begin{cases} k_x^r = \underline{\mathbf{k}}^r \cdot \underline{\mathbf{x}}_0 = k_1 \sin\theta^r \\ k_z^r = \underline{\mathbf{k}}^r \cdot \underline{\mathbf{z}}_0 = -k_1 \cos\theta^r \end{cases}$$

$$\underline{\mathbf{k}}^t = k_2 \underline{\beta_0^t} = k_x^t \underline{\mathbf{x}}_0 + k_z^t \underline{\mathbf{z}}_0$$

$$\begin{cases} k_x^t = \underline{\mathbf{k}}^t \cdot \underline{\mathbf{x}}_0 = k_2 \sin\theta^t \\ k_z^t = \underline{\mathbf{k}}^t \cdot \underline{\mathbf{z}}_0 = k_2 \cos\theta^t \end{cases}$$

$k_x{}^r = k_x{}^i$ ⇒

$$k_1 \sin\theta^r = k_1 \sin\theta^i$$
$$\theta^r = \theta^i$$

Legge della riflessione

$k_x{}^t = k_x{}^i$ ⇒

$$k_2 \sin\theta^t = k_1 \sin\theta^i$$

Legge di Snell o della rifrazione

apollonio@die.uniroma1.it

$$k_2 \sin\theta^t = k_1 \sin\theta^i \qquad per\ \mu_1 \cong \mu_2 \qquad \boxed{n_2 \sin\theta^t = n_1 \sin\theta^i}$$

$$\sin\theta^t = \frac{k_1}{k_2}\sin\theta^i = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta^i = \frac{\mu_1}{\mu_2}\sqrt{\frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}\sin\theta^i = \frac{\mu_1}{\mu_2}\frac{\varsigma_2}{\varsigma_1}\sin\theta^i$$

ammette soluzioni reali per: $\sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta^i \le 1 \qquad \sqrt{\varepsilon_1} \le \sqrt{\varepsilon_2} \Rightarrow n_1 \le n_2$

L'onda piana e uniforme polarizzata linearmente può essere decomposta in due onde piane ancora polarizzate linearmente:

$\underline{\mathbf{E}}_0^i$ →

$\underline{\mathbf{E}}_{0h}^i$ *diretto secondo* $\underline{\mathbf{y}}_0$ → $\underline{\mathbf{E}}_{0h}^i = \mathbf{E}_{0h}^i\,\underline{\mathbf{y}}_0$

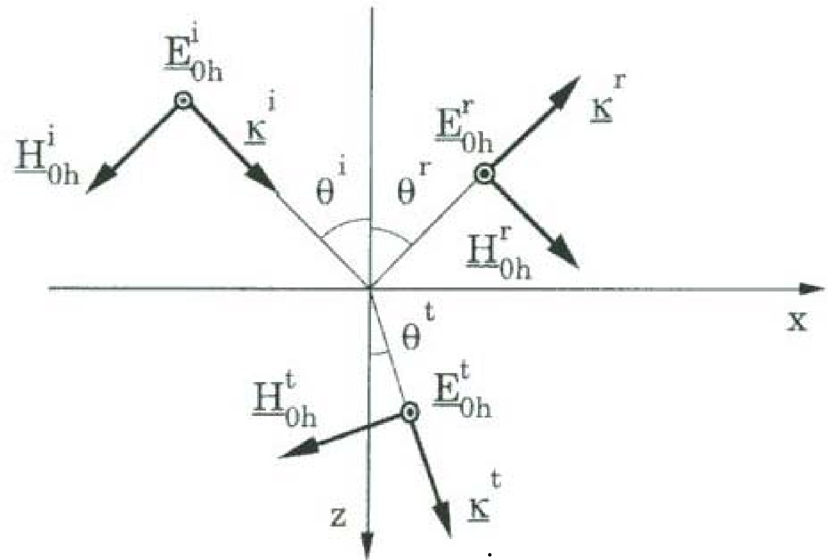
Polarizzazione orizzontale

$\underline{\mathbf{E}}_{0v}^i$ *contenuta nel piano x,z* → $\underline{\mathbf{H}}_{0v}^i = \mathbf{H}_{0v}^i\,\underline{\mathbf{y}}_0$

Polarizzazione verticale

apollonio@die.uniroma1.it

Polarizzazione orizzontale



$$\underline{\mathbf{H}}^i_{0h} = \frac{1}{\omega\mu_1}\underline{\mathbf{k}}^i \times \underline{\mathbf{E}}^i_{0h} = \frac{k_1}{\omega\mu_1}\left(\sin\theta^i \underline{\mathbf{x}}_0 + \cos\theta^i \underline{\mathbf{z}}_0\right) \times E^i_{0h}\underline{\mathbf{y}}_0 = \frac{E^i_{0h}}{\varsigma_1}\left(\sin\theta^i \underline{\mathbf{z}}_0 - \cos\theta^i \underline{\mathbf{x}}_0\right)$$

$$\underline{\mathbf{E}}^r_{0h} = E^r_{0h}\underline{\mathbf{y}}_0$$

$$\underline{\mathbf{E}}^t_{0h} = E^t_{0h}\underline{\mathbf{y}}_0$$

$$\underline{\mathbf{H}}^r_{0h} = \frac{1}{\omega\mu_1}\underline{\mathbf{k}}^r \times \underline{\mathbf{E}}^r_{0h} = \frac{k_1}{\omega\mu_1}\left(\sin\theta^r \underline{\mathbf{x}}_0 - \cos\theta^r \underline{\mathbf{z}}_0\right) \times E^r_{0h}\underline{\mathbf{y}}_0 = \frac{E^r_{0h}}{\varsigma_1}\left(\sin\theta^r \underline{\mathbf{z}}_0 + \cos\theta^r \underline{\mathbf{x}}_0\right)$$

$$\underline{\mathbf{H}}^t_{0h} = \frac{1}{\omega\mu_2}\underline{\mathbf{k}}^t \times \underline{\mathbf{E}}^t_{0h} = \frac{k_2}{\omega\mu_2}\left(\sin\theta^t \underline{\mathbf{x}}_0 + \cos\theta^t \underline{\mathbf{z}}_0\right) \times E^t_{0h}\underline{\mathbf{y}}_0 = \frac{E^t_{0h}}{\varsigma_2}\left(\sin\theta^t \underline{\mathbf{z}}_0 - \cos\theta^t \underline{\mathbf{x}}_0\right)$$

Per la condizione di continuità del <u>campo elettrico tangenziale</u> in z=0

$$\underline{\mathbf{z_0}} \times \left\{ \underline{\mathbf{E_0}}^{t} e^{-j\left(k_x{}^t x + k_y{}^t y\right)} - \left[\underline{\mathbf{E}}_0^i e^{-j\left(k_x^i x\right)} + \underline{\mathbf{E_0}}^{r} e^{-j\left(k_x{}^r x + k_y{}^r y\right)} \right] \right\} = 0$$

$$\underline{\mathbf{z_0}} \times \left\{ E_{0h}^t \underline{\mathbf{y}}_{\mathbf{0}} - \left[E_{0h}^i \underline{\mathbf{y}}_{\mathbf{0}} + E_{0h}^r \underline{\mathbf{y}}_{\mathbf{0}} \right] \right\} = 0 \quad \Rightarrow \quad E_{0h}^t - \left(E_{0h}^i + E_{0h}^r \right) = 0$$

Per la condizione di continuità del <u>campo magnetico tangenziale</u> in z=0

$$\underline{\mathbf{z_0}} \times \left\{ \underline{\mathbf{H_0}}^{t} e^{-j\left(k_x{}^t x + k_y{}^t y\right)} - \left[\underline{\mathbf{H}}_0^i e^{-j\left(k_x^i x\right)} + \underline{\mathbf{H_0}}^{r} e^{-j\left(k_x{}^r x + k_y{}^r y\right)} \right] \right\} = 0$$

$$\underline{\mathbf{z_0}} \times \left\{ \underline{\mathbf{H}}_{0h}^t - \left(\underline{\mathbf{H}}_{0h}^i + \underline{\mathbf{H}}_{0h}^r \right) \right\} = 0 \quad \Rightarrow \quad E_{0h}^t \frac{\cos\theta^t}{\varsigma_2} - \left(E_{0h}^i - E_{0h}^r \right) \frac{\cos\theta^i}{\varsigma_1} = 0$$

apollonio@die.uniroma1.it

$$\begin{cases} E_{0h}^{t} - \left(E_{0h}^{i} + E_{0h}^{r}\right) = 0 \\ E_{0h}^{t} \dfrac{\cos\theta^{t}}{\varsigma_2} - \left(E_{0h}^{i} - E_{0h}^{r}\right)\dfrac{\cos\theta^{i}}{\varsigma_1} = 0 \end{cases}$$

$$\Rightarrow \left(E_{0h}^{i} + E_{0h}^{r}\right)\frac{\cos\theta^{t}}{\varsigma_2} = \left(E_{0h}^{i} - E_{0h}^{r}\right)\frac{\cos\theta^{i}}{\varsigma_1}$$

$$S_{Eh} = \frac{E_{0h}^{r}}{E_{0h}^{i}} = \frac{\dfrac{\varsigma_2}{\varsigma_1}\cos\theta^{i} - \cos\theta^{t}}{\dfrac{\varsigma_2}{\varsigma_1}\cos\theta^{i} + \cos\theta^{t}}$$

coefficiente di riflessione per il campo elettrico

$$T_{Eh} = \frac{E_{0h}^{t}}{E_{0h}^{i}} = \frac{E_{0h}^{i} + E_{0h}^{r}}{E_{0h}^{i}} = 1 + S_{Eh}$$

coefficiente di trasmissione per il campo elettrico

$$S_{Hh} = \frac{H_{0h}^{r}}{H_{0h}^{i}} = \frac{\dfrac{1}{\varsigma_1}E_{0h}^{r}}{\dfrac{1}{\varsigma_1}E_{0h}^{i}} = S_{Eh}$$

coefficiente di riflessione per il campo magnetico

$$T_{Hh} = \frac{H_{0h}^{t}}{H_{0h}^{i}} = \frac{\dfrac{1}{\varsigma_2}E_{0h}^{t}}{\dfrac{1}{\varsigma_1}E_{0h}^{i}} = \frac{\varsigma_1}{\varsigma_2}\frac{E_{0h}^{t}}{E_{0h}^{i}} = \frac{\varsigma_1}{\varsigma_2}T_{Eh}$$

coefficiente di trasmissione per il campo magnetico

apollonio@die.uniroma1.it

Se $\theta^i \neq 0$ *e ricordando che* $\sin\theta^t = \frac{\mu_1}{\mu_2}\frac{\varsigma_2}{\varsigma_1}\sin\theta^i$ *e* $\mu_1 \cong \mu_2$

$$S_{Eh} = \frac{\frac{\mu_2 \sin\theta^t}{\mu_1 \sin\theta^i}\cos\theta^i - \cos\theta^t}{\frac{\mu_2 \sin\theta^t}{\mu_1 \sin\theta^i}\cos\theta^i + \cos\theta^t} \cong \frac{\sin\theta^t \cos\theta^i - \cos\theta^t \sin\theta^i}{\sin\theta^t \cos\theta^i + \cos\theta^t \sin\theta^i} = \frac{\sin\left(\theta^t - \theta^i\right)}{\sin\left(\theta^t + \theta^i\right)}$$

che non si annulla per nessun valore di $\theta^i \leq \frac{\pi}{2}$ *se* $\theta^i \neq \theta^t$ *cioè se i due mezzi sono diversi*

Polarizzazione verticale

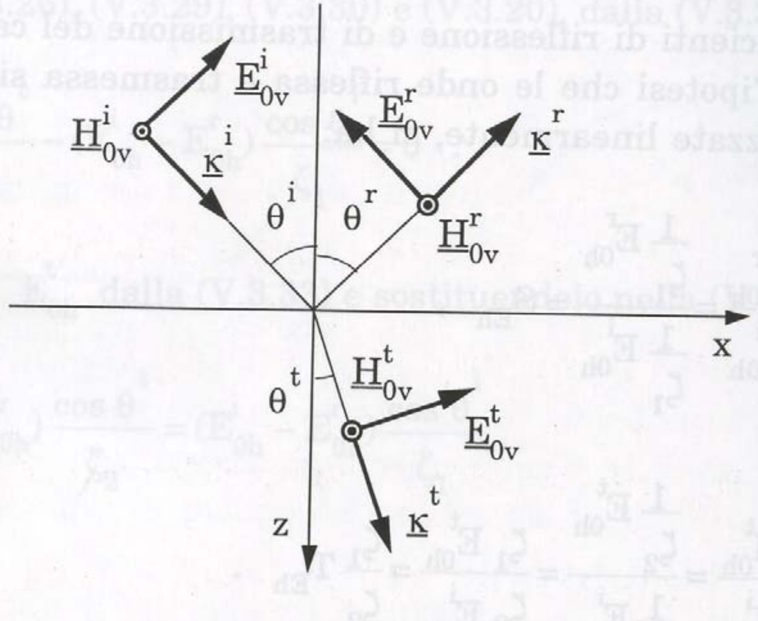


Fig. V.6

$\underline{\mathbf{E}}^i_{0v}$ *contenuta nel piano x,z* $\qquad \underline{\mathbf{H}}^i_{0v} = \mathbf{H}^i_{0v}\,\underline{\mathbf{y}}_0$

$$\underline{\mathbf{E}}^i_{0v} = -\frac{1}{\omega\varepsilon_1}\underline{\mathbf{k}}^i \times \underline{\mathbf{H}}^i_{0v} = -\frac{k_1}{\omega\varepsilon_1}\left(\sin\theta^i\,\underline{\mathbf{x}}_0 + \cos\theta^i\,\underline{\mathbf{z}}_0\right)\times H^i_{0v}\,\underline{\mathbf{y}}_0 = -\varsigma_1 H^i_{0v}\left(\sin\theta^i\,\underline{\mathbf{z}}_0 - \cos\theta^i\,\underline{\mathbf{x}}_0\right)$$

$$\underline{\mathbf{H}}^r_{0v} = H^r_{0v}\,\underline{\mathbf{y}}_0$$

$$\underline{\mathbf{H}}^t_{0v} = H^t_{0v}\,\underline{\mathbf{y}}_0$$

apollonio@die.uniroma1.it

$$\underline{\mathbf{E}}_{0v}^{r} = -\frac{1}{\omega\varepsilon_1}\underline{\mathbf{k}}^{r}\times\underline{\mathbf{H}}_{0v}^{r} = -\frac{k_1}{\omega\varepsilon_1}\left(\sin\theta^{r}\,\underline{\mathbf{x}}_0 - \cos\theta^{r}\,\underline{\mathbf{z}}_0\right)\times H_{0v}^{r}\,\underline{\mathbf{y}}_0 = -\varsigma_1 H_{0h}^{r}\left(\sin\theta^{r}\,\underline{\mathbf{z}}_0 + \cos\theta^{r}\,\underline{\mathbf{x}}_0\right)$$

$$\underline{\mathbf{E}}_{0v}^{t} = -\frac{1}{\omega\varepsilon_2}\underline{\mathbf{k}}^{t}\times\underline{\mathbf{H}}_{0v}^{t} = -\frac{k_2}{\omega\varepsilon_2}\left(\sin\theta^{t}\,\underline{\mathbf{x}}_0 + \cos\theta^{t}\,\underline{\mathbf{z}}_0\right)\times H_{0v}^{t}\,\underline{\mathbf{y}}_0 = -\varsigma_2 H_{0v}^{t}\left(\sin\theta^{t}\,\underline{\mathbf{z}}_0 - \cos\theta^{t}\,\underline{\mathbf{x}}_0\right)$$

Per la condizione di continuità del <u>campo magnetico tangenziale</u> in z=0

$$\underline{\mathbf{z}}_{\mathbf{0}}\times\left\{H_{0v}^{t}\,\underline{\mathbf{y}}_{\mathbf{0}} - \left[H_{0v}^{i}\,\underline{\mathbf{y}}_{\mathbf{0}} + H_{0v}^{r}\,\underline{\mathbf{y}}_{\mathbf{0}}\right]\right\} = 0 \quad \Rightarrow \quad H_{0v}^{t} - \left(H_{0v}^{i} + H_{0v}^{r}\right) = 0$$

Per la condizione di continuità del <u>campo elettrico tangenziale</u> in z=0

$$\underline{\mathbf{z}}_{\mathbf{0}}\times\left\{\underline{\mathbf{E}}_{0v}^{t} - \left(\underline{\mathbf{E}}_{0v}^{i} + \underline{\mathbf{E}}_{0v}^{r}\right)\right\} = 0 \quad \Rightarrow \quad \varsigma_2 H_{0v}^{t}\cos\theta^{t} - \varsigma_1\cos\theta^{i}\left(H_{0v}^{i} - H_{0v}^{r}\right) = 0$$

$$\begin{cases} H_{0v}^{t} - \left(H_{0v}^{i} + H_{0v}^{r}\right) = 0 \\ \varsigma_2 H_{0v}^{t}\cos\theta^{t} - \varsigma_1\cos\theta^{i}\left(H_{0v}^{i} - H_{0v}^{r}\right) = 0 \end{cases}$$

apollonio@die.uniroma1.it

$$S_{Hv} = \frac{H_{0v}^{r}}{H_{0v}^{i}} = \frac{\cos\theta^{i} - \dfrac{\varsigma_2}{\varsigma_1}\cos\theta^{t}}{\cos\theta^{i} + \dfrac{\varsigma_2}{\varsigma_1}\cos\theta^{t}}$$

coefficiente di riflessione per il campo magnetico (polarizzazione verticale)

$$T_{Hv} = \frac{H_{0v}^{t}}{H_{0v}^{i}} = \frac{H_{0v}^{i} + H_{0v}^{r}}{H_{0v}^{i}} = 1 + S_{Hv}$$

coefficiente di trasmissione per il campo magnetico (polarizzazione verticale)

$$S_{Ev} = \frac{E_{0v}^{r}}{E_{0v}^{i}} = \frac{\varsigma_1 H_{0v}^{r}}{\varsigma_1 H_{0v}^{i}} = S_{Hv}$$

coefficiente di riflessione per il campo elettrico (polarizzazione verticale)

$$T_{Ev} = \frac{E_{0v}^{t}}{E_{0v}^{i}} = \frac{\varsigma_2 H_{0v}^{t}}{\varsigma_1 H_{0v}^{i}} = \frac{\varsigma_2}{\varsigma_1} T_{Hv}$$

coefficiente di trasmissione per il campo elettrico (polarizzazione verticale)

apollonio@die.uniroma1.it

Casi particolari (1)

Se $\theta^i \neq 0$ *e ricordando che* $\sin\theta^t = \frac{\mu_1}{\mu_2}\frac{\varsigma_2}{\varsigma_1}\sin\theta^i$ *e* $\mu_1 \cong \mu_2$

$$S_{Ev} = \frac{\cos\theta^i - \frac{\mu_2 \sin\theta^t}{\mu_1 \sin\theta^i}\cos\theta^t}{\cos\theta^i + \frac{\mu_2 \sin\theta^t}{\mu_1 \sin\theta^i}\cos\theta^i} \cong \frac{\sin\theta^i \cos\theta^i - \cos\theta^t \sin\theta^t}{\sin\theta^i \cos\theta^i + \cos\theta^t \sin\theta^t} = \frac{\sin\left(2\theta^i\right) - \sin\left(2\theta^t\right)}{\sin\left(2\theta^i\right) + \sin\left(2\theta^t\right)} =$$

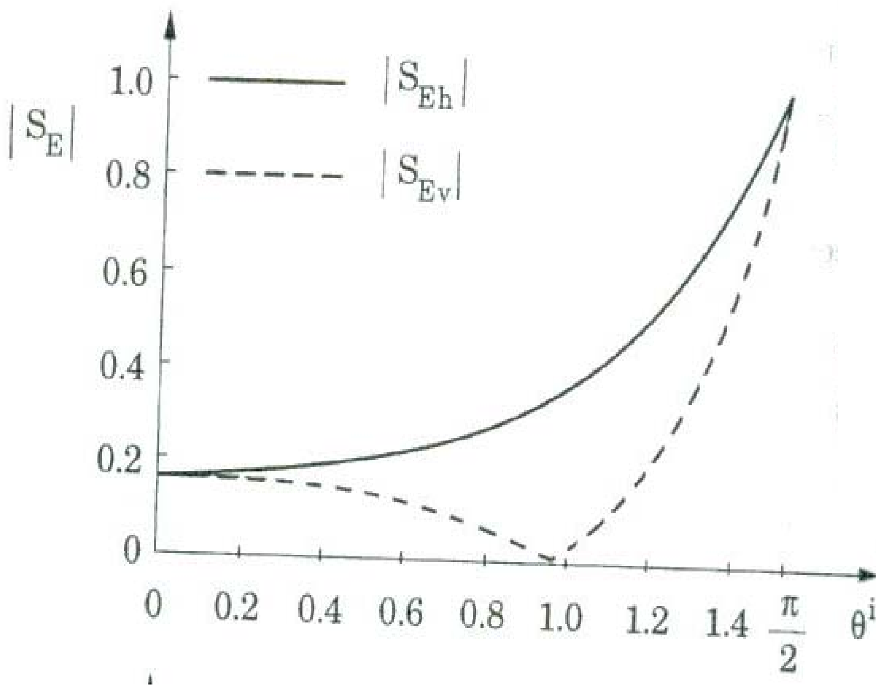
$$= \frac{\cos\left(\theta^i + \theta^t\right)\sin\left(\theta^i - \theta^t\right)}{\sin\left(\theta^i + \theta^t\right)\cos\left(\theta^i - \theta^t\right)} = \frac{\tan\left(\theta^i - \theta^t\right)}{\tan\left(\theta^i + \theta^t\right)}$$

A differenza del caso di polarizzazione orizzontale, in questo caso esiste un valore di $\theta^i \neq \theta^t$ *per cui risulta* $S_{Ev} = 0$

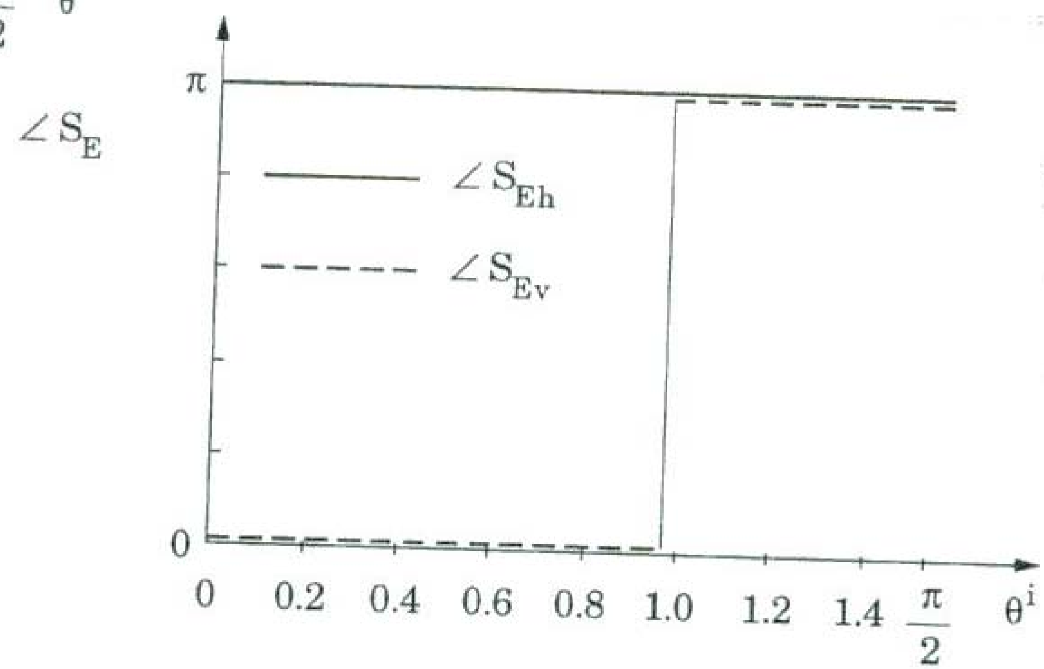
$$\theta^i + \theta^t = \frac{\pi}{2} \Rightarrow \theta^t = \frac{\pi}{2} - \theta^i \qquad \frac{\sin\theta^i}{\sin\theta^t} = \frac{\sin\theta^i}{\cos\theta^i} = \tan\theta^i = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}\sin\theta^i \cong \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\boxed{\theta^i = \arctan\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

angolo di Brewster ->
trasmissione totale



|S_E|
1.0
0.8
0.6
0.4
0.2
0
|S_Eh|
|S_Ev|
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
π/2
θ^i



∠S_E
π
0
∠S_Eh
∠S_Ev
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
π/2
θ^i

Casi particolari (2)

$$k_2 \sin\theta^t = k_1 \sin\theta^i$$ *ripartendo dalla legge di Snell*

$$\sin\theta^t = \frac{k_1}{k_2}\sin\theta^i = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta^i = \frac{\mu_1}{\mu_2}\sqrt{\frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}\sin\theta^i = \frac{\mu_1}{\mu_2}\frac{\varsigma_2}{\varsigma_1}\sin\theta^i$$

$$\sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta^i \le 1$$ *se* $\mu_1\varepsilon_1 \le \mu_2\varepsilon_2$ *è certamente soddisfatta*

se $\mu_1\varepsilon_1 > \mu_2\varepsilon_2 \Rightarrow n_1 > n_2 \Rightarrow \theta^t > \theta^i$

Al crescere di θ^i *si raggiungerà un valore* θ_L^i *per cui risulta* $\theta^t = \dfrac{\pi}{2}$

$$\sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta_L^i = 1 \quad \Longrightarrow \quad \theta_L^i = \arcsin\sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}$$ *angolo limite*

apollonio@die.uniroma1.it

Per $\theta^i > \theta^i_L$ *l'onda trasmessa non è piana e uniforme*

Facciamo l'ipotesi che l'onda trasmessa sia piana e non uniforme (evanescente):

$$\underline{\mathbf{E}}^t(x,y,z) = \underline{\mathbf{E_0}}^t e^{-j\underline{\mathbf{k}}^t \cdot \underline{\mathbf{r}}} = \underline{\mathbf{E_0}}^t e^{-j\left(k_x{}^t x + k_y{}^t y + k_z{}^t z\right)}$$

$$\underline{\mathbf{k}}^t = \underline{\boldsymbol{\beta}}^t - j\,\underline{\boldsymbol{\alpha}}^t \qquad \underline{\boldsymbol{\beta}}^t \cdot \underline{\boldsymbol{\alpha}}^t = 0$$

$$\beta^{t\,2} - \alpha^{t\,2} = k_2^2 = \omega^2 \mu_2 \varepsilon_2$$

$$k_x{}^t = k_x{}^i \quad \Rightarrow \beta_x{}^t - j\alpha_x{}^t = k_1 \sin\theta^i \Rightarrow \beta_x{}^t = k_1 \sin\theta^i$$

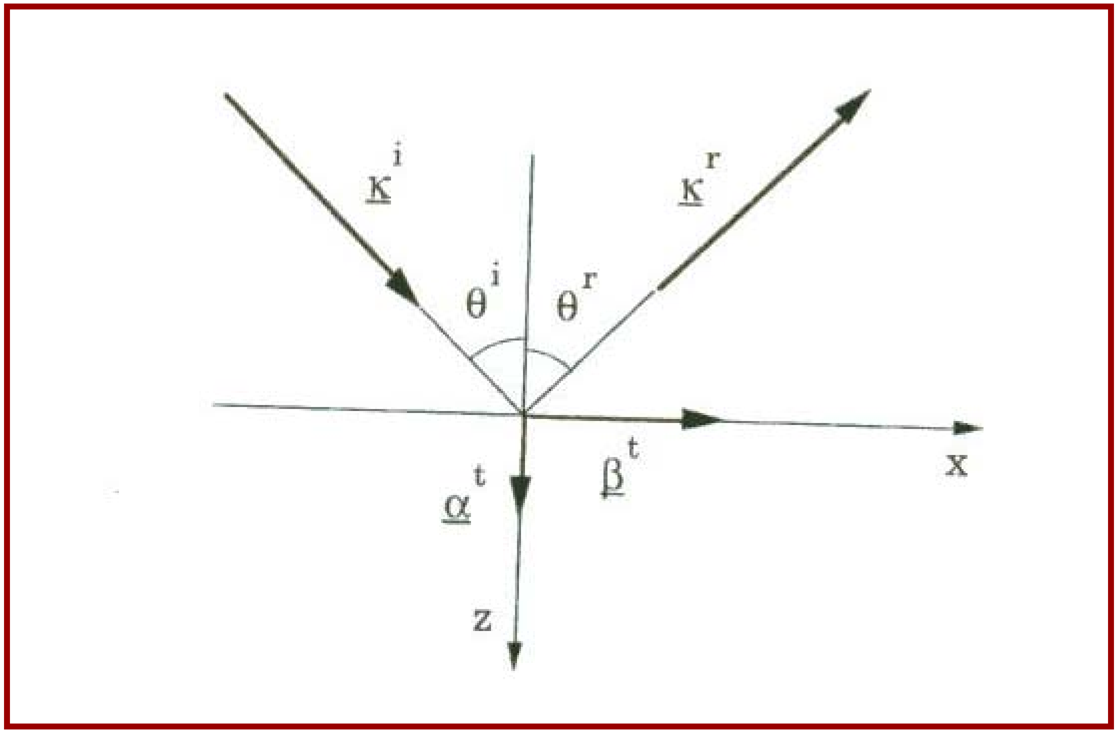
$$\Rightarrow \alpha_x{}^t = 0$$

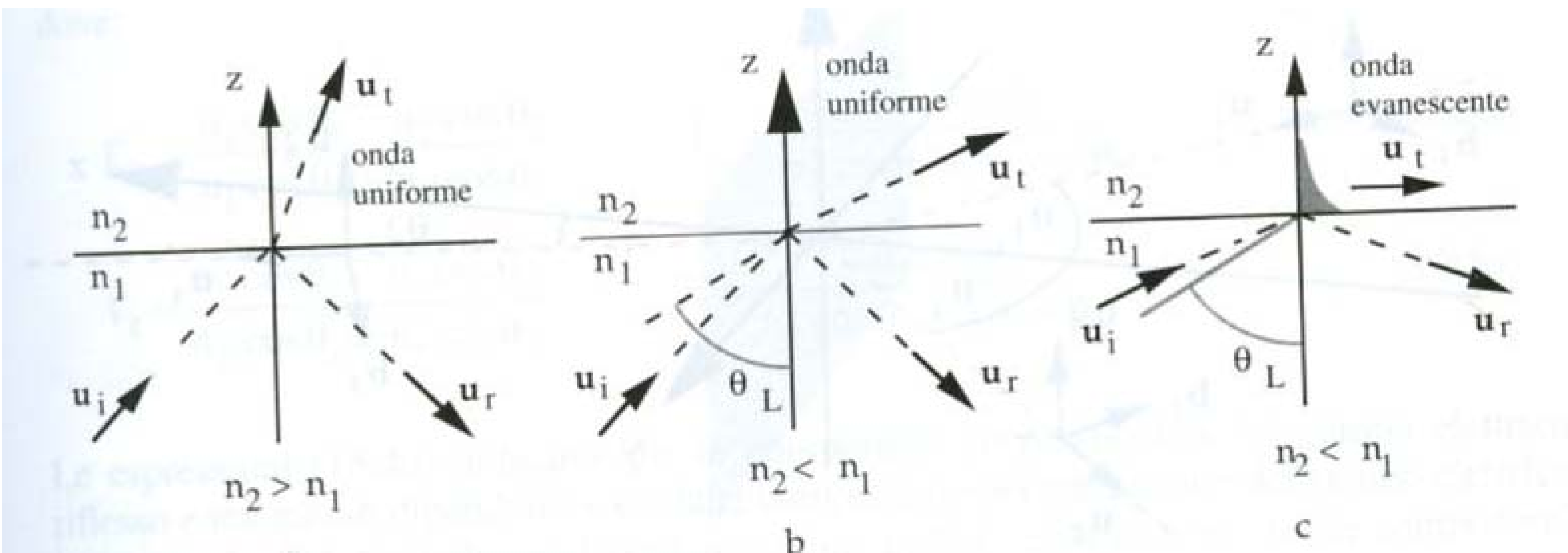
$$k_y{}^t = 0 \quad \Rightarrow \beta_y{}^t - j\alpha_y{}^t = 0 \quad \Rightarrow \beta_y{}^t = 0$$

$$\Rightarrow \alpha_y{}^t = 0$$

$$\boxed{\underline{\boldsymbol{\beta}}^t = \beta^t\,\underline{\mathbf{x}}_0 \qquad \underline{\boldsymbol{\alpha}}^t = \alpha^t\,\underline{\mathbf{z}}_0}$$

apollonio@die.uniroma1.it


κ^i
κ^r
θ^i
θ^r
x
α^t
β^t
z


z
u t
onda
uniforme
n2
n1
ui
ur
n2 > n1
z
onda
uniforme
ut
n2
n1
ui
θ L
ur
n2 < n1
b
z
onda
evanescente
u t
n2
n1
ui
ur
θ L
n2 < n1
c

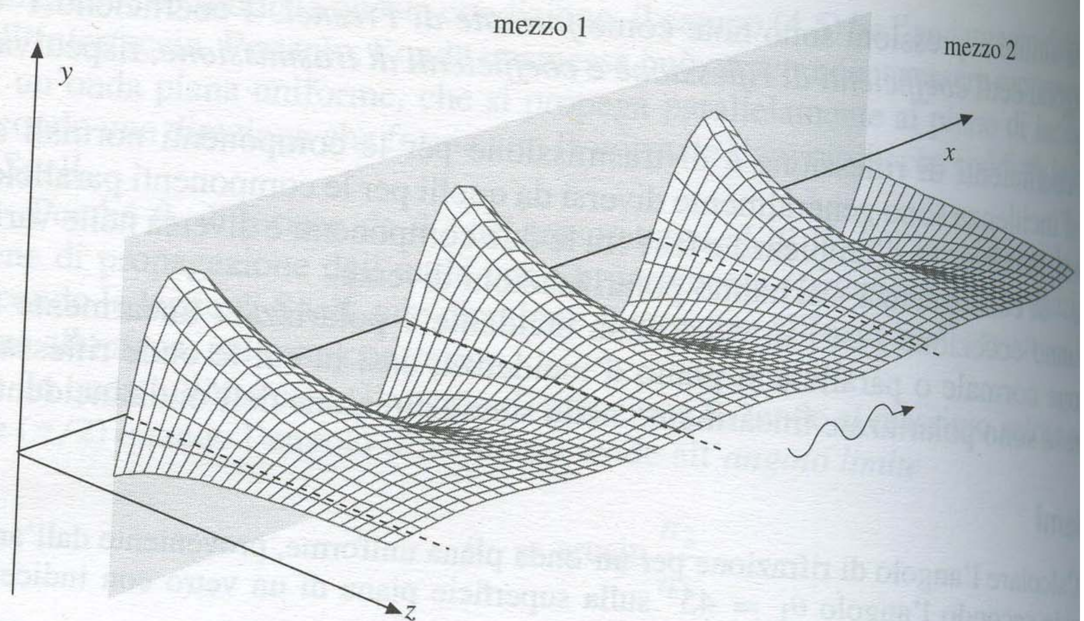


Figura 4.2 Onda evanescente che si propaga lungo l'interfaccia.

apollonio@die.uniroma1.it

Incidenza obliqua sulla superficie di un buon conduttore

Per l'<u>onda riflessa</u> possiamo supporre che sia piana e uniforme $\theta^r = \theta^i$

Per l'<u>onda rifratta (o trasmessa)</u> dovrà risultare $\underline{\boldsymbol{\alpha}}^t \neq 0$ *inoltre per la continuità dei campi tangenziali in z=0, essendo* $\underline{\boldsymbol{\alpha}}^i = \underline{\boldsymbol{\alpha}}^r = 0$

$$\underline{\boldsymbol{\alpha}}^t = \alpha^t \underline{\mathbf{z}}_0$$

D'altronde se $\theta^i \neq 0$ *sarà* $\beta_x^t \neq 0$ *quindi sembra che l'onda non sia piana e uniforme*

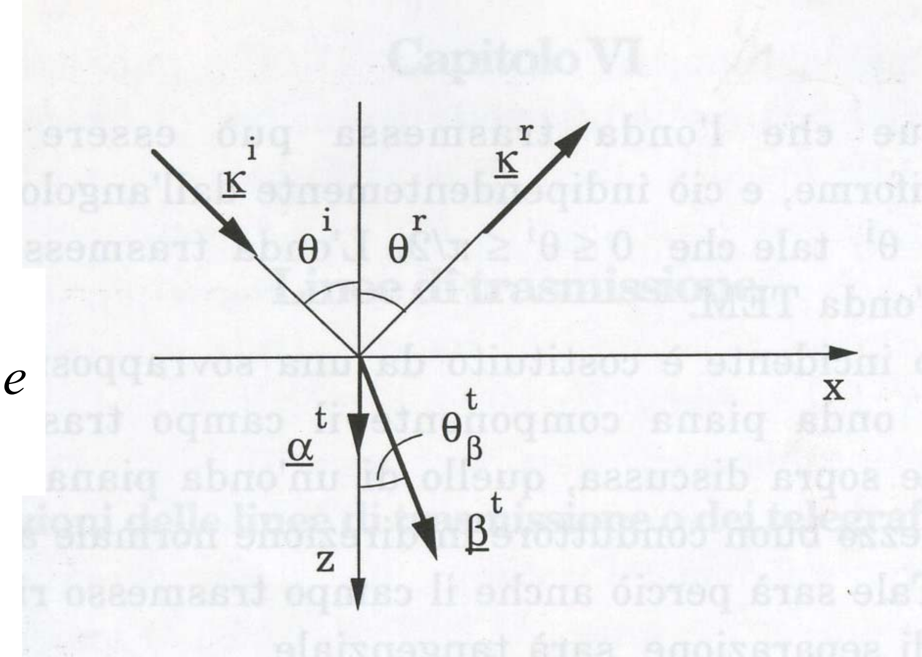


Fig. V.10

apollonio@die.uniroma1.it

Ricordando che:

$$k^2 = \mathbf{k}\cdot\mathbf{k} = \omega^2\mu\varepsilon_c = \omega^2\mu\varepsilon - j\omega\mu\sigma$$

$$(\boldsymbol{\beta} - j\boldsymbol{\alpha})\cdot(\boldsymbol{\beta} - j\boldsymbol{\alpha}) = \beta^2 - \alpha^2 - 2j\boldsymbol{\beta}\cdot\boldsymbol{\alpha} = k^2 = \omega^2\mu\varepsilon - j\omega\mu\sigma$$

$$\beta^2 - \alpha^2 = \omega^2\mu\varepsilon \qquad \boldsymbol{\beta}\cdot\boldsymbol{\alpha} = \frac{\omega\mu\sigma}{2}$$

a) $$\beta^{t^2} - \alpha^{t^2} = \omega^2\mu_2\varepsilon_2$$

$$\beta^t\alpha^t\cos\theta^t_\beta = \frac{\omega\mu_2\sigma_2}{2}$$

b) $k^t_x = k^i_x \Rightarrow \beta^t\sin\theta^t_\beta = k_1\sin\theta^i$

dalla a)

$$\beta^t > \alpha^t \rightarrow \beta^t\alpha^t\cos\theta^t_\beta < \beta^t\alpha^t < \beta^{t^2} \rightarrow \beta^t > \sqrt{\frac{\omega\mu_2\sigma_2}{2}}$$

apollonio@die.uniroma1.it

dalla b)

$$\sin\theta_\beta^t = \frac{k_1 \sin\theta^i}{\beta^t} < \frac{\omega\sqrt{\mu_1\varepsilon_1}}{\sqrt{\frac{\omega\mu_2\sigma_2}{2}}}\sin\theta^i = \sqrt{\frac{2\omega\mu_1\varepsilon_1}{\mu_2\sigma_2}}\sin\theta^i = \sqrt{\frac{2\mu_1}{\mu_2}}\sqrt{\frac{\omega\varepsilon_1}{\sigma_2}}\sin\theta^i$$

$\mu_1 \cong \mu_2$ $\quad \sin\theta^i \le 1$ $\quad \sigma_2 >> \omega\varepsilon_1$ ➡ $\sin\theta_\beta^t << 1$ ➡ $\theta_\beta^t \cong 0$

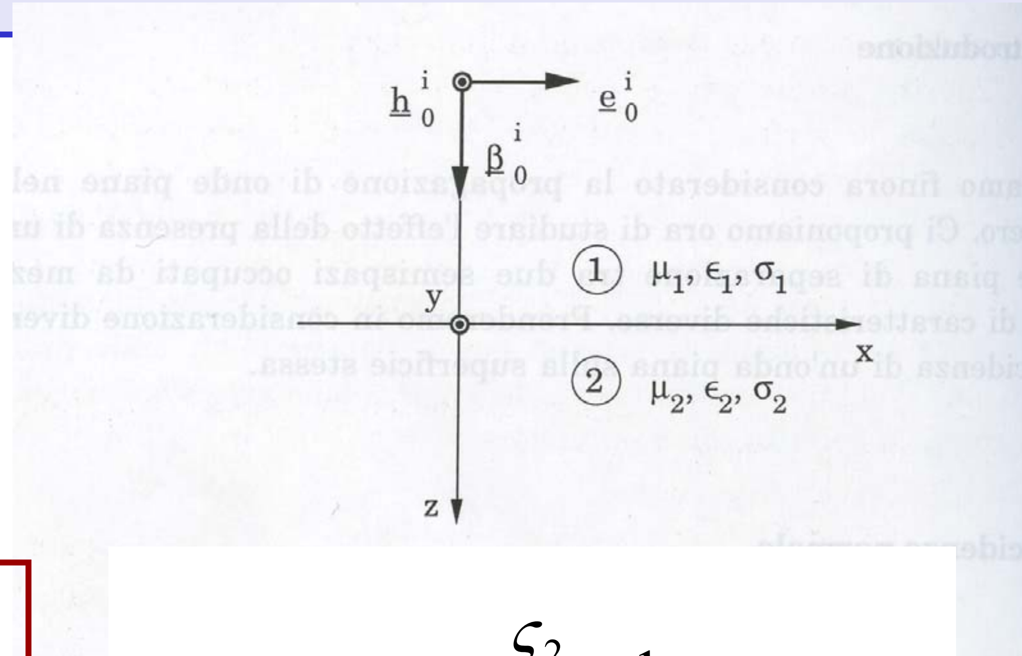
L'onda è praticamente piana e uniforme -> TEM

apollonio@die.uniroma1.it

Riflessione e rifrazione: incidenza normale

$$\theta^i = 0$$

$$\theta^t = 0$$



$$S_{Eh} = \frac{E_{0h}^r}{E_{0h}^i} = \frac{\frac{\varsigma_2}{\varsigma_1}\cos\theta^i - \cos\theta^t}{\frac{\varsigma_2}{\varsigma_1}\cos\theta^i + \cos\theta^t}$$

$$T_{Eh} = \frac{E_{0h}^t}{E_{0h}^i} = \frac{E_{0h}^i + E_{0h}^r}{E_{0h}^i} = 1 + S_{Eh}$$

$$S_{Eh} = \frac{E_{0h}^r}{E_{0h}^i} = \frac{\frac{\varsigma_2}{\varsigma_1} - 1}{\frac{\varsigma_2}{\varsigma_1} + 1} = \frac{\varsigma_2 - \varsigma_1}{\varsigma_2 + \varsigma_1}$$

apollonio@die.uniroma1.it

Casi particolari

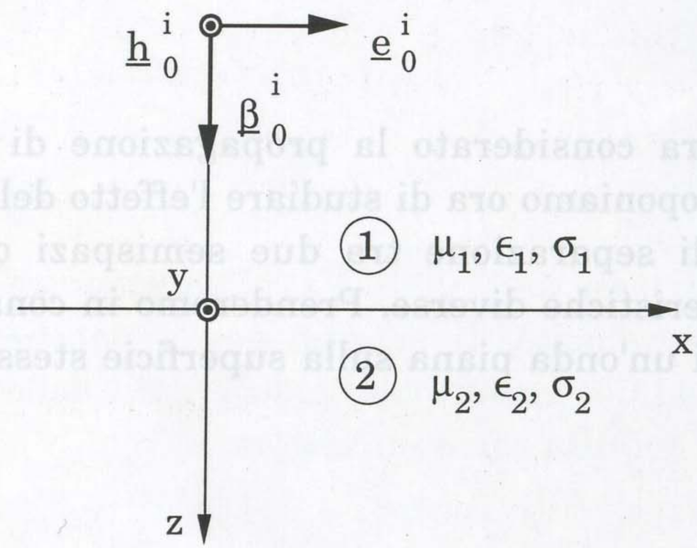


Fig. V.1

dielettrico non disssipativo

$$\sigma_1 = 0$$

Buon conduttore

$$\sigma_2 >> \omega\varepsilon_2$$

$$\sigma_2 >> \omega\varepsilon_1 \qquad \mu_2 \cong \mu_1$$

$$\varsigma_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

$$\varsigma_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} \cong \sqrt{\frac{j\omega\mu_1}{\sigma_2}} = \sqrt{j}\sqrt{\frac{\omega\mu_1}{\sigma_2}} =$$

$$= \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\sqrt{\frac{\omega\mu_1}{\sigma_2}} = (1+j)\sqrt{\frac{\omega\mu_1}{2\sigma_2}}$$

$$\left|\frac{\varsigma_2}{\varsigma_1}\right| = \left|(1+j)\sqrt{\frac{\omega\varepsilon_1}{2\sigma_2}}\right| = \sqrt{\frac{\omega\varepsilon_1}{\sigma_2}} << 1$$

$$S_E = \Gamma = \frac{\frac{\varsigma_2}{\varsigma_1} - 1}{\frac{\varsigma_2}{\varsigma_1} + 1} \Rightarrow -1$$

$$T_E = T = 1 + \Gamma = \frac{2\frac{\varsigma_2}{\varsigma_1}}{\frac{\varsigma_2}{\varsigma_1} + 1} \Rightarrow 0$$

Non è lecito eseguire il passaggio al limite per $\sigma_2 \to \infty$ *quindi considero il mezzo 1 con* $\sigma_2 = 0$ *ed il mezzo 2 con* $\sigma_2 \to \infty$

Nella regione 2: E=H=0

$$\mathbf{n} \times \left(\mathbf{H}^+ - \mathbf{H}^-\right) = J_S$$

La condizione di continuità dei campi elettrici e magnetici tangenziali in z=0

$$-\underline{\mathbf{z}}_0 \times \left[\underline{\mathbf{H}_0}^i + \underline{\mathbf{H}_0}^r e^{-j\left(k_x{}^r x + k_y{}^r y\right)}\right] = \underline{\mathbf{J}}_S$$ *densità superficiale di corrente*

$$-\underline{\mathbf{z}}_0 \times \left[\underline{\mathbf{E}_0}^i + \underline{\mathbf{E}_0}^r e^{-j\left(k_x{}^r x + k_y{}^r y\right)}\right] = 0$$

$$0 = E_0^i + E_0^r \Rightarrow E_0^r = -E_0^i$$

$$S_E = -1$$

apollonio@die.uniroma1.it

$$-\underline{\mathbf{z}}_{\mathbf{0}} \times \left[\underline{\mathbf{H}}_{\mathbf{0}}{}^{i} + \underline{\mathbf{H}}_{\mathbf{0}}{}^{r} e^{-j\left(k_x{}^r x + k_y{}^r y\right)} \right] = \underline{\mathbf{J}}_S$$

$$H_0^r = \frac{E_0^r}{\varsigma_1} = -\frac{E_0^i}{\varsigma_1} = -H_0^i$$

$$\underline{\mathbf{H}}_0^r = -H_0^r \underline{\mathbf{y}}_{\mathbf{0}} = H_0^i \underline{\mathbf{y}}_{\mathbf{0}} = \underline{\mathbf{H}}_0^i$$

$$k_x{}^r = 0 \quad k_y{}^r = 0$$

$$\underline{\mathbf{J}}_S = -\underline{\mathbf{z}}_{\mathbf{0}} \times \left[\underline{\mathbf{H}}_{\mathbf{0}}{}^{i} + \underline{\mathbf{H}}_{\mathbf{0}}{}^{r} \right] = -2\underline{\mathbf{z}}_{\mathbf{0}} \times \underline{\mathbf{H}}_0^i = 2H_0^i \underline{\mathbf{x}}_{\mathbf{0}} = 2\frac{E_0^i}{\varsigma_1} \underline{\mathbf{x}}_{\mathbf{0}}$$

Nella regione 1:

$$\underline{\mathbf{E}}_1(z) = \underline{\mathbf{E}}^i(z) + \underline{\mathbf{E}}^r(z) = E_0^i \underline{\mathbf{x}}_0 \left(e^{-jk_1 z} - e^{jk_1 z}\right) = -2jE_0^i \underline{\mathbf{x}}_0 \sin(k_1 z)$$

$$\underline{\mathbf{H}}_1(z) = \underline{\mathbf{H}}^i(z) + \underline{\mathbf{H}}^r(z) = H_0^i \underline{\mathbf{y}}_0 \left(e^{-jk_1 z} + e^{jk_1 z}\right) = 2H_0^i \underline{\mathbf{y}}_0 \cos(k_1 z) = 2\frac{E_0^i}{\varsigma_1} \underline{\mathbf{y}}_0 \cos(k_1 z)$$

*Il campo totale rappresenta **un'onda stazionaria** in quanto la sua fase non varia con le coordinate (non è un'onda che si propaga).*

apollonio@die.uniroma1.it

Il campo istantaneo:

$$\underline{\mathbf{E}}_1(z,t) = \mathrm{Re}\left[\underline{\mathbf{E}}_1(z)e^{j\omega t}\right] = 2\left|E_0^i\right|\underline{\mathbf{x}}_0 \sin(k_1\, z)\sin(\omega t + \varphi)$$

$$\underline{\mathbf{H}}_1(z,t) = \mathrm{Re}\left[\underline{\mathbf{H}}_1(z)e^{j\omega t}\right] = 2\frac{\left|E_0^i\right|}{\varsigma_1}\underline{\mathbf{y}}_0 \cos(k_1\, z)\cos(\omega t + \varphi)$$

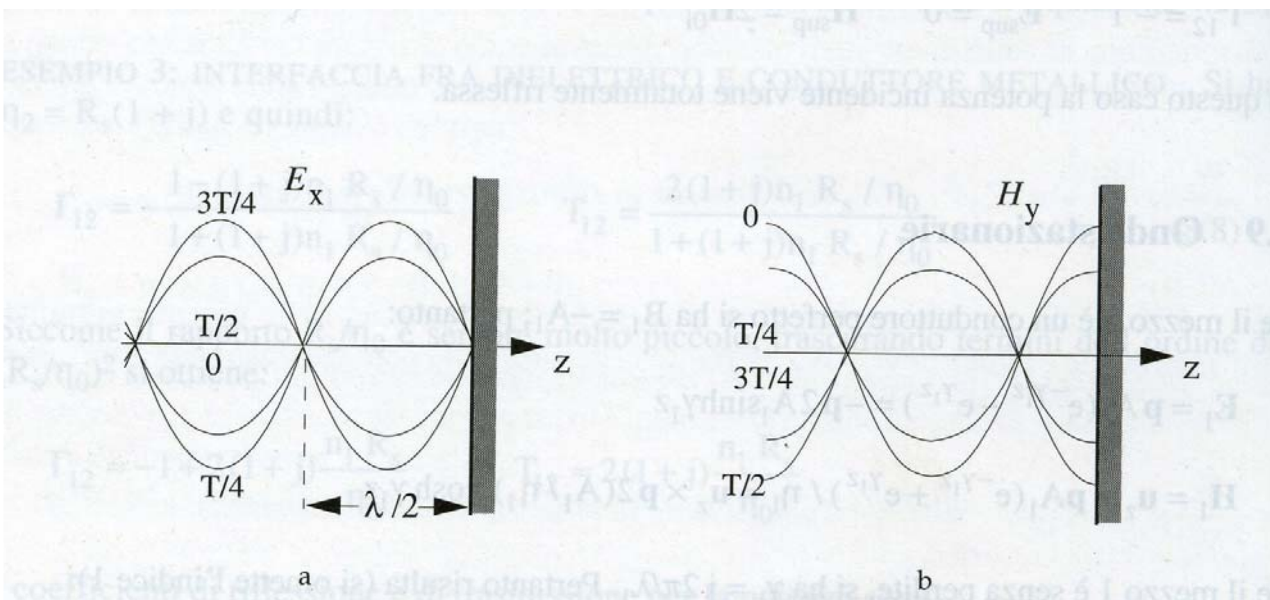


Figura 9.1